Exam - Solid State Physics I

Wednesday, 2nd November 2016, 09:00 – 12:00

This is a closed-book exam. You are not allowed to bring books, notes etc. You can use a basic or scientific calculator, but no other electronic equipment with capabilities to display or pronounce the course content.

Do not forget to indicate your full name and student number on ***each*** *sheet. Please write in a clear way!*

There are 5 problems with total points of 100.

1) Crystal structure (15 points).

a. A single layer of carbon atoms called graphene forms a two-dimensional (2D) honeycomb lattice as shown in figure 1. Find the Bravais lattice for graphene. Draw the basis and primitive lattice vectors (you are allowed to draw directly on Fig. 1). **(5 pt.)**

b. Stacking graphene layers together forms three-dimensional (3D) single crystal called graphite as shown in figure 2. What is the Bravais lattice in this ABA-type stacking? Give your argument. **(4 pt.)**

c. For the primitive cell (the parallelepiped with green edges) shown in Figure 2, calculate how many carbon atoms are there in the primitive cell of graphite? **(6 pt.)**

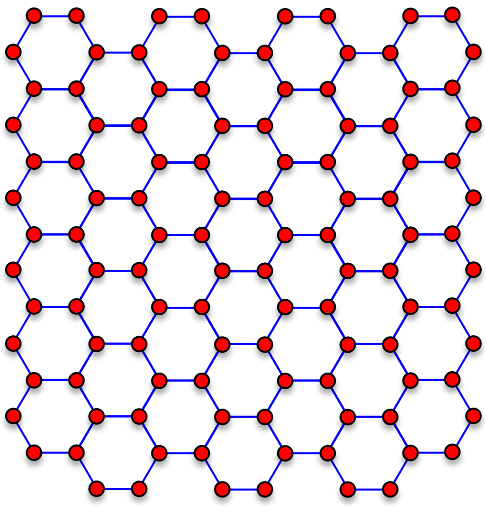


Figure 1

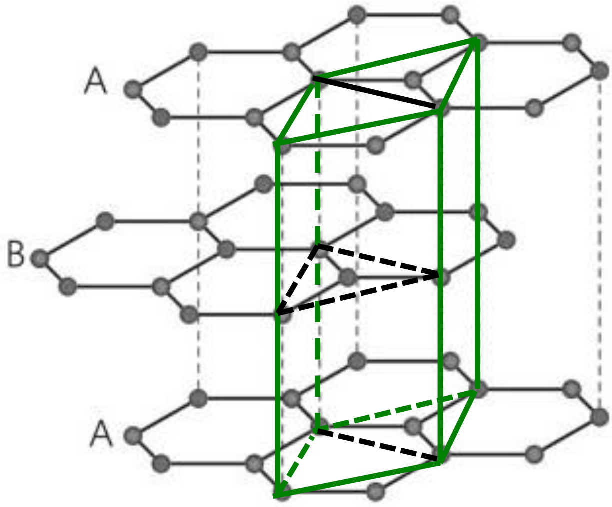


Figure 2

2) X-Ray diffraction in crystals (20 points).

There exist hollow molecules entirely made of carbon atoms, the most famous one is $\boldsymbol{C_{60}}$, a molecule made of 60 carbon atoms arranged in the form of a icosahedron: that is, $\boldsymbol{C_{60}}$ has roughly the shape of a sphere. $\boldsymbol{C_{60}}$ molecules crystallize in an ***FCC*** lattice of charge-neutral molecules with cubic lattice constant $a = 14.11$ Å.

a. What kind of cohesive force keeps the molecules together in this crystal? **(2 pt.)**

b. We know from experiment that in a $\boldsymbol{C_{60}}$ molecule the distance from the each carbon nuclei to the center of the molecule is $R = 3.5$ Å. Assume that 360 electrons are uniformly distributed on the surface of $\boldsymbol{C_{60}}$ molecule, such that the electron density can be written as $n(\boldsymbol{r}) = A\,\delta(|\boldsymbol{r}| - R)$, where the δ is a delta function. Determine the constant A (surface area of a sphere: $4\pi R^2$, where R is the radius of the sphere) **(5 pt.)**

c. Determine the *atomic form factor* $f_{\boldsymbol{G}}$ as a function of the reciprocal lattice vector $\boldsymbol{G}$. **(5 pt.)**

d. Calculate the *Structure Factor* for the $(2\ 0\ 0)$ and $(1\ 1\ 1)$ planes. Explain from your result why, experimentally, the $(2\ 0\ 0)$ X-ray diffraction peak is much weaker compared to the $(1\ 1\ 1)$ peak. **(8 pt.)**

Hint:

Structure factor for ***FCC*** lattice is

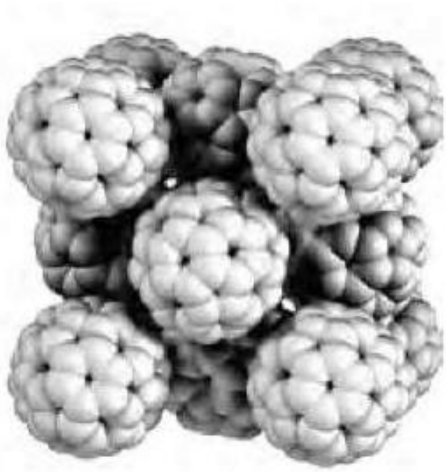
$$S_{\boldsymbol{G}} = f_{\boldsymbol{G}}\left(1 + e^{-i\pi(h+k)} + e^{-i\pi(k+l)} + e^{-i\pi(h+l)}\right)$$

Atomic form factor is

$$f_{\boldsymbol{G}} = 4\pi \int_0^{\infty} dr \cdot n(r) \cdot r^2 \cdot \frac{sin(\boldsymbol{G}\cdot\boldsymbol{r})}{\boldsymbol{G}\cdot\boldsymbol{r}}$$

and for delta function,

$$\int_a^b \delta(x-c) f(x) dx = \begin{cases} f(c) & if\ c \in [a;b] \\ 0 & if\ c \notin [a;b] \end{cases}$$



3) Thermal properties of metal (20 points).

a. Calculate 3D density of states $D(\omega)$ for phonons in a cubic crystal with size $L \times L \times L$. Use Debye approximation for dispersion relation $\omega = vk$. **(6 pt.)**

b. Calculate the number of phonons n_{ph} in this crystal. Show its dependence on temperature T in the two limiting cases: when $k_B T \gg \hbar\omega$ and when $k_B T \ll \hbar\omega$. **(6 pt.)**

Hint: $\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx$

c. Thermal conductivity coefficient in metal is given by $k \approx k_e = \frac{1}{3} C_e v l = \frac{\pi^2 N k_B^2}{3m} \tau T$.

Here electron scattering rate mainly due to interaction with phonons or impurities: $\frac{1}{\tau} = \frac{1}{\tau_{ph}} + \frac{1}{\tau_i} + \cdots$. Sketch k as a function of T. **(4 pt.)**

d. Also discuss and sketch how electrical conductivity in metal depends on temperature T. **(4 pt.)**

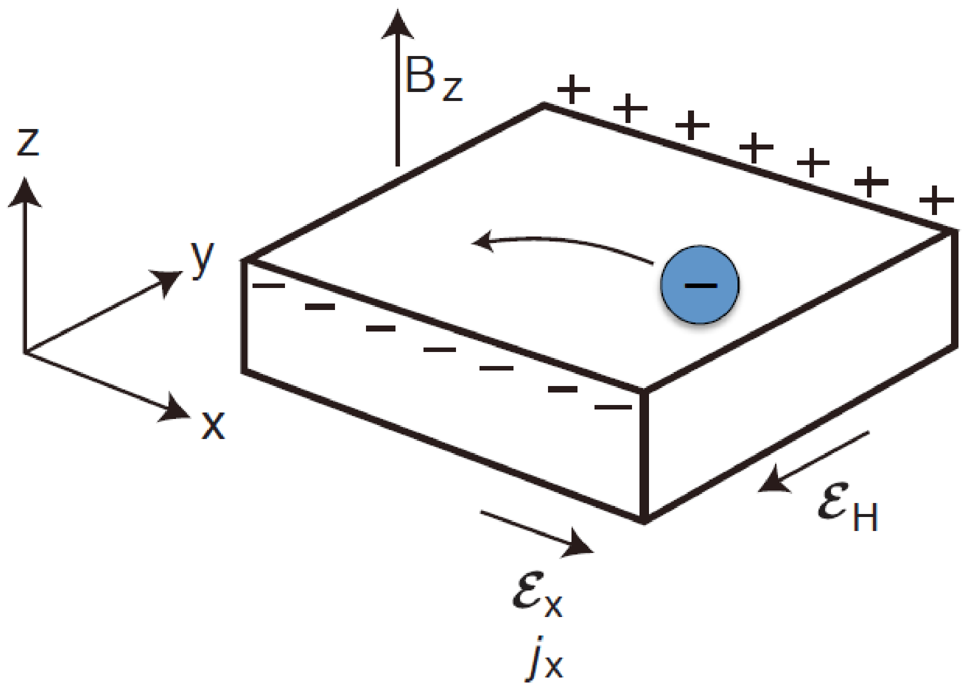
4) Hall effect (25 points).

a. Consider a conducting specimen (see figure) in a longitudinal electric field E_x and a transverse magnetic field $B = B_z$. Write down equations of motion of electrons in this specimen, and find steady state solutions for electron velocities in all three dimensions. **(6 pt.)** Hint: Lorentz force on an electron is $\boldsymbol{F} = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

b. The Hall coefficient is defined by $R_H = \frac{E_y}{j_x B}$, derive the expression for R_H as a function of electron concentration n. **(8 pt.)**

c. Now assume there are two types of carriers in this specimen, electrons and holes, like the case of a semiconductor material. Show $R_H = \frac{1}{e} \cdot \frac{p - nb^2}{(p+nb)^2}$, where n is electron concentration, p is hole concentration, and $b = \frac{\mu_e}{\mu_h}$ is the electron-hole mobility ratio. **(8 pt.)**

d. What do you expect the R_H will be for a material where $m_e = m_h$ and $p = n$? **(3 pt.)**



5) Semiconductor and superconductor properties (20 points).

a. Describe the difference between *insulator*, *semiconductor*, *semimetal* and *metal*. **(4 pt.)**

b. $p - n$ junction **(10 pt.)**

i. Explain what are *p* and *n* type semiconductors,

ii. Draw the schematic energy diagram.

iii. What will happen on the interface if you now bring them in contact?

iv. Draw the energy levels at the interface before and after diffusive equilibrium is established.

v. Based on the energy diagram explain how a solar cell works.

c. Qualitatively explain the physical phenomena shown on the picture below. Why the heavy load can be held by a magnet on a superconductor? **(6 pt.)**